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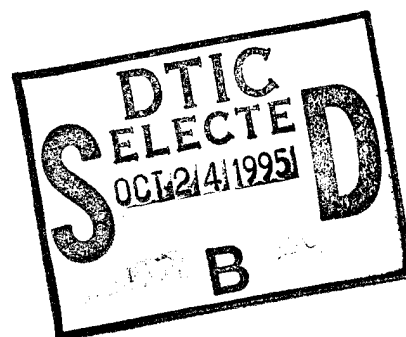
APPLICATION OF THE HILL EQUATION IN LONG DISTANCE INTERCEPTION

by

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APPLICATION OF THE HILL EQUATION IN LONG DISTANCE INTERCEPTION*

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ABSTRACT

A method to reduce the model error of the Hill equation for

long relative distances is proposed in this paper; this is done

by coordinate shifting to reduce the long relative distance of

the original system to a small one in a new system. That is, as

the relative distance of the interceptor reaches a particular

precision parameter value ρ_1 (ρ_1 is defined as a controlled

precision parameter, for example, $\rho_1 = 500km$), an imaginary moving

reference system with its origin at the interceptor's initial

position is set up, and the relative motion parameters of the

interceptor in its original system are converted into those in

the new system (obviously the initial relative distance in the

new system will be $\vec{\rho_{(0)}} = 0$) . As a result, the long distance

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problem becomes a small one, and the descriptive precision of the Hill equation is improved. This paper provides a theoretical analysis for this method and two examples of computer simulation. This method does indeed reduce model error for long relative distances in the Hill equation, producing satisfactory results.

With the development of modern aeronautic technology, spacecraft must engage in a variety of motion and flight duties, such as fixed time target interception. The problem of fixed time interception can be studied in inertial reference systems ([1] introduces the classic Herrick method, Godal method, and Lambert flight time theorem, etc.). This problem can also be studied from an appropriate motion reference system, and reference [1] offered a description of two Hill equations for relative spacecraft motion.

Inertial motion system research is based on orbital dynamics theory, and a precise solution for the fixed intercept time Δt is found through an integrated motion equation or an iterative solution of the algebraic transcendental equation. One cannot directly write out the analytical solution's results, so it is inconvenient to analyze the problem. Motion systems in the Hill equation make it possible to derive a relative motion equation

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through gravitational linearization; it is superior in that it is easy to derive a solution which closely approximates the analytical solution. But it only has a highly precise solution for short relative distances. When the relative distance is longer (for example, when $\rho > 1000km$), the model error in the Hill equation is relatively large, and the precision of the solution is decreased.

In actual engineering problems, sooner or later we always encounter situations where spacecraft flight time (and, therefore relative distance) is relatively long, such as in the case of long-distance interception. In the initial study of the problem, we hope to have a fast and convenient methodology such as the Hill equation, but if we then use the simple Hill equation, the error results caused by model error are intolerable.

The critical point in reducing model error is in whether or not we can make a long relative distance problem into a small relative distance problem through the use of some type of linear conversion (single or multiple). Assuming the interceptor is released from a satellite, after time t_1 its relative distance $\rho = \rho_1$ (ρ_1 being a precise control parameter, for example $\rho_1 = 500km$), and constructing an imaginary satellite target trajectory coordinate system (motion reference system) at this point and at this time, then at this specific moment the interceptor is at relative position $\vec{\rho_0} = 0$ in this new system.

Size of the interceptor's distance, we can always make the long relative distance into a limited small relative distance through a limited frequency of this kind of target coordinate system conversion, so that every time we use the Hill equation we can guarantee $\rho < \rho_1$, and reduce its model error.

1. The Hill Equation

In a distance-squared inverted gravitational field, the satellite revolves around trajectory I. The satellite orbit system O-XYZ is the target motion coordinate system. If the spacecraft is moving close to the satellite, then next to its orbital path there is a small bias error movement. At time t the geocentric vector is $\vec{r_2}$, and the satellite location vector $\vec{r_1}$ is relative to the satellite location vector $\vec{\rho} = \vec{r_2} - \vec{r_1}$. This is shown in Fig. 1. If the spacecraft and the satellite are both subject to the function of the distance square inverted central force gravitational field, then:

$$\frac{d^2\vec{\rho}}{dt^2} = \frac{d^2\vec{r_2}}{dt^2} - \frac{d^2\vec{r_1}}{dt^2} = -\frac{\mu\vec{r_2}}{r_2^3} + \frac{\mu\vec{r_1}}{r_1^3} = \mu\,\frac{\vec{r_1} - \left(\frac{r_1}{r_2}\right)^3\vec{r_2}}{r_1^3}$$

If $\vec{\dot{\omega}}$ and $\vec{\omega}$ are the motion coordinate rotation angle of acceleration and angle of speed, we can obtain the vector form relative motion equation as:

$$\frac{\delta^2\vec{\rho}}{dt^2} + 2\left(\vec{\omega} \times \frac{\delta\vec{\rho}}{dt}\right) + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \vec{\dot{\omega}} \times \vec{\rho} = \mu\,\frac{\vec{r_1} - \left(\frac{r_1}{r_2}\right)^3\vec{r_2}}{r_1^3}$$

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With target coordinates $(X, Y, Z)^T$ in the spacecraft motion system 0-XYZ, we can use gravitational error linearization to obtain the spacecraft relative motion differential equation (assuming $\rho \ll r_1$):

$$\begin{cases} \ddot{X} = 3n^2X + 2n\dot{Y} \\ \ddot{Y} = - 2n\dot{X} \\ \ddot{Z} = - n^2Z \end{cases} \tag{1}$$

n in the equation is the trajectory path angular speed of the satellite, $n = |\vec{\omega}| = \sqrt{\mu/r_1^3}$, $\mu = fM = 3.986005 \times 10^{14}$ m^3/s^2 is the gravitational constant, and M is earth's mass. This group of constant coefficient differential equations for linear integration approximates a description of the relative motion of two spacecraft, and is called the Hill equation. The smaller the relative distance ρ, the more precise the description.

making $\qquad \vec{\rho} = (X, Y, Z,)^T; \quad \vec{V} = (\dot{X}, \dot{Y}, \dot{Z})^T$

Integrating Eq. (1) yields the spacecraft relative motion parameter at time τ:

$$\begin{bmatrix} \rho_\tau \\ V_\tau \end{bmatrix} = \begin{bmatrix} \phi_{rr}(\tau) & \phi_{rv}(\tau) \\ \phi_{vr}(\tau) & \phi_{vv}(\tau) \end{bmatrix} \begin{bmatrix} \rho_{(0)} \\ V_{(0)} \end{bmatrix} \tag{2}$$

$\rho_{(0)}$ and $V_{(0)}$ in the equation are the initial parameters for the spacecraft at time $\tau = 0$, and $\phi_{rr}, \phi_{rv}, \phi_{vr}, \phi_{vv}$ are the relative motion

condition shift matrices.

$$\phi_{\rho\rho}(\tau) = \begin{pmatrix} 4 - 3\cos n\tau & 0 & 0 \\ 6(\sin n\tau - n\tau) & 1 & 0 \\ 0 & 0 & \cos n\tau \end{pmatrix}$$

$$\phi_{\rho\nu}(\tau) = \begin{pmatrix} \dfrac{\sin n\tau}{n} & \dfrac{2(1 - \cos n\tau)}{n} & 0 \\ \dfrac{-2(1 - \cos n\tau)}{n} & \dfrac{4\sin n\tau}{n} - 3\tau & 0 \\ 0 & 0 & \dfrac{\sin n\tau}{n} \end{pmatrix}$$

$$\phi_{\nu\rho}(\tau) = \begin{pmatrix} 3n\sin n\tau & 0 & 0 \\ 6n(\cos n\tau - 1) & 0 & 0 \\ 0 & 0 & -n\sin n\tau \end{pmatrix}$$

$$\phi_{\nu\nu}(\tau) = \begin{pmatrix} \cos n\tau & 2\sin n\tau & 0 \\ -2\sin n\tau & 4\cos n\tau - 3 & 0 \\ 0 & 0 & \cos n\tau \end{pmatrix}$$

Through the linearization of gravitational error, the Hill equation yields an analytical solution. From Eq. (1) we can see that relative motion possesses the following special characteristics: from it we can separate solutions for two mutually independent movements, the longitudinal plane (XOY) and lateral (for direction Z).

2. Target Coordinate Conversion and Relative Parameter Conversion

To simplify the problem, this paper will only study the longitudinal problem at present and not consider the lateral (direction Z) movement, and continue to present an analysis based on a three-dimensional vector form (Z direction value is assumed at 0).

Suppose the satellite is in motion in orbit 1, and we release a flight craft with relative speed of $\vec{V}_0$. The motion system at time $\tau=0$ is O-XYZ, and the satellite latitude is u_0. After time τ_1, O-XYZ follows the satellite to fly to position O'-X'Y'Z'. At this time, the traveling craft's relative position vector is $\vec{\rho}$, and let us assume that at this time $\rho=\rho_1$, as shown in Fig. 2.
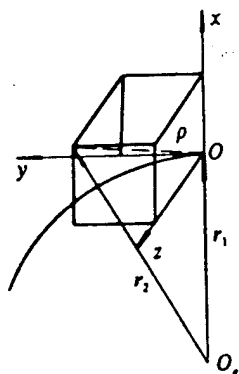
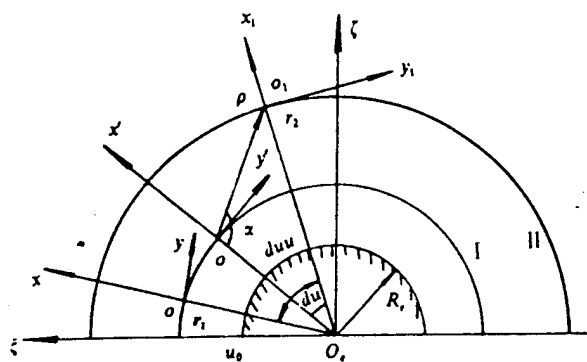Fig. 1. Satellite trajectory system   Fig. 2. Target coordinate conversion

At O_1, we construct an imaginary motion reference system O_1-$X_1Y_1Z_1$, and we can easily know

$$r_1 = Re + H$$

$$n_{S_1} = \sqrt{\frac{\mu}{r_1^3}}$$

$$\angle xO_eX' = n_{S_1}\tau_1$$

$$r_2^2 = r_1^2 + \rho^2 - 2r_1\rho\cos\alpha$$

Re in the equation is the earth's radius, and H is the orbital height of the satellite.

Mark that at time τ_1, the traveling craft's relative motion parameters are $\vec{\rho}$, and $\vec{V}$ in O'-X'Y'Z', then we have

$$\vec{\rho}_{(\tau_1)} = \phi_\rho(\tau_1)\vec{V}_{(0)}; \qquad \vec{V}_{(\tau_1)} = \phi_w(\tau_1)\vec{V}_0$$

Assume that the three components of $\vec{\rho}_{r1}$ in O'-X'Y'Z' are $\rho_{(1,1)}$, $\rho_{(2,1)}$, and $\rho_{(3,1)}$, then

$$\alpha = \frac{\pi}{2} + \text{arctg}[\rho_{(1,1)}/\rho_{(3,1)}]$$

$$du = \text{sign}[\rho_{(2,1)}]\arccos\frac{r_1^2 + r_2^2 - \rho^2}{2r_1r_2}$$

$$duu = du + n_{S_1}\tau_1$$

Below, we convert the traveling craft's relative parameters $\vec{\rho}$ and $\vec{V}$ in the original system to the relative parameters $\vec{\rho}_{1(0)}$ and $\vec{V}_{1(0)}$ in the new system. Obviously, $\vec{\rho}_{1(0)}=0$ and $\vec{V}_{1(0)}$ can be derived as follows:

Take Oe-$\xi\eta\zeta$ as an inertial system, from

$$\vec{V}_e = \vec{V}_{r1} + \vec{V}_{e1} = \vec{V}_{r2} + \vec{V}_{e2}$$

to obtain

$$B_{(u_0 + du)} \begin{bmatrix} 0 \\ n_{S1} r_2 \\ 0 \end{bmatrix} + B_{(u_0 + u_{S1} r_1)} \vec{V}_{r1}$$

$$= B_{(u_0 + du)} \begin{bmatrix} 0 \\ n_{S2} r_2 \\ 0 \end{bmatrix} + B_{(u_0 + du)} \vec{V}_{1(0)} \qquad (3)$$

In the equation

$$n_{S2} = \sqrt{\mu / r_1^3}$$

$$B_{(du)} = \begin{bmatrix} \cos du & -\sin du & 0 \\ \sin du & \cos du & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$B_{(du)}$ is the target coordinate shift matrix between $O'-X'Y'Z'$ and $O_1-X_1Y_1Z_1$.

From Eq. (3), we simplify to obtain

$$\vec{V}_{1(0)} = B_{(du)}^{-1} \vec{V}_{r1} + \begin{bmatrix} 0 \\ (n_{S1} - n_{S2}) r_2 \\ 0 \end{bmatrix} \qquad (4)$$

This is the method of expressing the relative speed in the new system.

3. Two Functional Examples

We use a satellite-released interceptor terminating a missile in an unpowered missile trajectory as an example. The satellite orbit path is a polar orbit, at a height of H=640km. Time of interceptor release is $\tau=0$, latitude is $u_0=50°$.

The missile is launched due north, and its termination point motion parameters are $H_{\bar{K}}=170$km, speed $V_{\bar{K}}=7390$m/s, local-speed angle of inclination $\Theta_{\bar{K}}=20.6°$, and latitude $\phi_{\bar{K}}=90^0$. Set the interceptor's largest attack impact value at 6421m/s, and the longest interception time at 1500s. The satellite orbit path plane and missile launch plane match, and perform a unidirectional chase intercept, without calculating earth flatness, or considering earth rotation. This paper offers three sets of computer simulation results: the first set is based on a direct calculation of the Hill equation (indicated as H), the second uses the calculated results from an inertial system as a standard parameter (indicated as HO), and the third offers the method in this paper for calculation results (indicated as HA), and graphs the deviation curve of HO with respect to H and HA.

Example 1 sets intercept time at $\tau=1500$s and uses the above-mentioned three types of methods to find the intercept target's needed attack impact speed increase ΔV. Of these, the HA simulation process precision control parameter $\rho_1=200$km, and the target coordinate system shift frequency is n (times). The results are as follows:

$$HO: \Delta V = (-1506.77, 1022.12, 0)^T$$
$$H: \Delta V = (-1448.03, 1062.59, 0)^T$$

Note that δV_1 is the H deviation, $\delta V_1 = 71.33$m/s, δV_2 is the HA deviation, the δV_1, $\delta V_2 \sim \rho_1$ and $n \sim \rho_1$ curves are as shown in Figs. 3 and 4.

Example 2 sets the intercept attack impact speed increase as follows: $\Delta V = (-1000, 1000, 0)^T$ m/s, obtaining $\rho_1 = 200$km. The intercept termination point target coordinate (error distance) follows the change in time τ (and relative distance ρ) as shown in Fig. 5 (H, HA). The HA coordinate shift frequency curve $n \sim \rho$ is shown in Fig. 6.
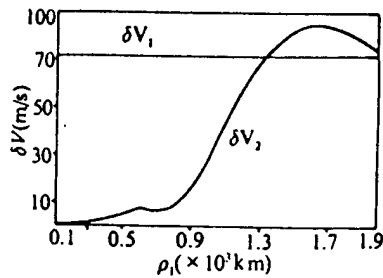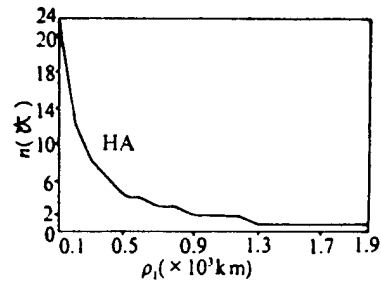
Fig. 3. $\delta V \sim \rho_1$ curve
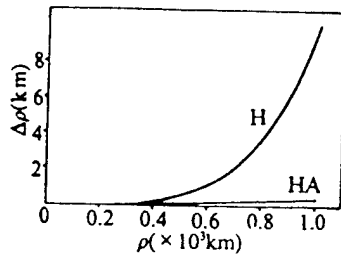
Fig. 4. $n \sim \rho_1$ curve

Fig. 5. Δρ~ρ curve

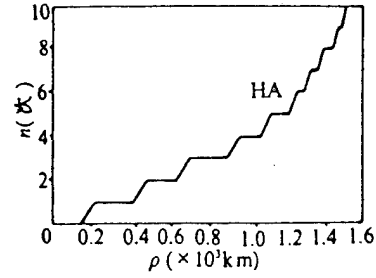
Fig. 6. n~ρ curve

4. Conclusions

(1) HA can effectively reduce the model error in the Hill equation for long relative distances, and increase its descriptive precision. Figs. 3 and 5 show satisfactory results.

(2) The precision control parameters ρ_1 for HA can generally be 100~500km, and at this time can guarantee adequate precision of the Hill equation for long-distance problems.

(3) HA does not need numerical differentials or iterated solution algebraic transcendental equations; one only needs to borrow the Hill equation for an analytical solution and limited frequency linear conversion, and does not need to consider the type of interceptor trajectory curve, making it easy to program in-flight processing.

(4) HA has a definite practical application value, and can

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be broadly used for problems such as interception targets, non-orbital motion, orbital shift, and orbital path intersection, etc.

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